

BIG IDEAS

The **concept of a limit** is foundational to calculus.

Differential calculus develops the concept of **instantaneous rate of change**.

Integral calculus develops the concept of determining a product involving a **continuously changing** quantity over an interval.

Derivatives and integrals are **inversely related**.

Learning Standards

Curricular Competencies	Content
<i>Students are expected to do the following:</i> Reasoning and modelling - Develop thinking strategies to solve puzzles and play games - Explore, analyze, and apply mathematical ideas using reason, technology, and other tools Estimate reasonably and demonstrate fluent, flexible, and strategic thinking about number Model with mathematics in situational contexts Think creatively and with curiosity and wonder when exploring problems Understanding and solving - Develop, demonstrate, and apply conceptual understanding of mathematical ideas through play, story, inquiry, and problem solving Visualize to explore and illustrate mathematical concepts and relationships - Apply flexible and strategic approaches to solve problems - Solve problems with persistence and a positive disposition - Engage in problem-solving experiences connected with place, story, cultural practices, and perspectives relevant to local First Peoples communities, the local community, and other cultures	<i>Students are expected to know the following:</i> functions and graphs limits: - left and right limits - limits to infinity - continuity differentiation: rate of change differentiation rules - higher order, implicit applications integration: approximations - fundamental theorem of calculus methods of integration applications

Learning Standards (continued)

Curricular Competencies	Content
Communicating and representing • Explain and justify mathematical ideas and decisions in many ways • Represent mathematical ideas in concrete, pictorial, and symbolic forms • Use mathematical vocabulary and language to contribute to discussions in the classroom • Take risks when offering ideas in classroom discourse Connecting and reflecting • Reflect on mathematical thinking • Connect mathematical concepts with each other, other areas, and personal interests • Use mistakes as opportunities to advance learning • Incorporate First Peoples worldviews, perspectives, knowledge, and practices to make connections with computer science concepts	

Big Ideas – Elaborations

- **concept of a limit:**

- Differentiation and integration are defined using limits.

Sample questions to support inquiry with students:

- Why is a limit useful?
- How can we use historical examples (e.g., Achilles and the tortoise) to describe a limit?

- **instantaneous rate of change:**

- developing rate of change from average to instantaneous

Sample questions to support inquiry with students:

- How can a rate of change be instantaneous?
- When do we use rate of change?

- **continuously changing:**

- area (height x width) under a curve where the height of the region is changing; volume of a solid (area x length) where cross-sectional area is changing; work (force x distance) where force is changing
- Finding these products requires finding an infinite sum.

Sample questions to support inquiry with students:

- What is the value of using rectangles to approximate the area under a curve?
- Why is the fundamental theorem of calculus so fundamental?

- **inversely related:**

- The fundamental theorem of calculus describes the relationship between integrals and antiderivatives.

Sample questions to support inquiry with students:

- How are derivatives and integrals related?
- Why are antiderivatives important?
- What is the difference between an antiderivative and an integral?

Curricular Competencies – Elaborations

- **thinking strategies:**
 - using reason to determine winning strategies
 - generalizing and extending
- **analyze:**
 - examine the structure of and connections between mathematical ideas (e.g., limits, derivatives, integrals)
- **reason:**
 - inductive and deductive reasoning
 - predictions, generalizations, conclusions drawn from experiences (e.g., with puzzles, games, and coding)
- **technology:**
 - graphing technology, dynamic geometry, calculators, virtual manipulatives, concept-based apps
 - can be used for a wide variety of purposes, including:
 - exploring and demonstrating mathematical relationships
 - organizing and displaying data
 - generating and testing inductive conjectures
 - mathematical modelling
- **other tools:**
 - manipulatives such as algebra tiles and other concrete materials
- **Estimate reasonably:**
 - be able to defend the reasonableness of an estimate across mathematical contexts
- **fluent, flexible, and strategic thinking:**
 - includes:
 - using known facts and benchmarks, partitioning, applying number strategies to approximate limits, derivatives, and integrals
 - choosing from different ways to think of a number or operation (e.g., Which will be the most strategic or efficient?)
- **Model:**
 - use mathematical concepts and tools to solve problems and make decisions (e.g., in real-life and/or abstract scenarios)
 - take a complex, essentially non-mathematical scenario and figure out what mathematical concepts and tools are needed to make sense of it
- **situational contexts:**
 - including real-life scenarios and open-ended challenges that connect mathematics with everyday life
- **Think creatively:**
 - by being open to trying different strategies
 - refers to creative and innovative mathematical thinking rather than to representing math in a creative way, such as through art or music

Curricular Competencies – Elaborations

- **curiosity and wonder:**
 - asking questions to further understanding or to open other avenues of investigation
- **inquiry:**
 - includes structured, guided, and open inquiry
 - noticing and wondering
 - determining what is needed to make sense of and solve problems
- **Visualize:**
 - create and use mental images to support understanding
 - Visualization can be supported using dynamic materials (e.g., graphical relationships and simulations), concrete materials, drawings, and diagrams.
- **flexible and strategic approaches:**
 - deciding which mathematical tools to use to solve a problem
 - choosing an effective strategy to solve a problem (e.g., guess and check, model, solve a simpler problem, use a chart, use diagrams, role-play)
- **solve problems:**
 - interpret a situation to identify a problem
 - apply mathematics to solve the problem
 - analyze and evaluate the solution in terms of the initial context
 - repeat this cycle until a solution makes sense
- **persistence and a positive disposition:**
 - not giving up when facing a challenge
 - problem solving with vigour and determination
- **connected:**
 - through daily activities, local and traditional practices, popular media and news events, cross-curricular integration
 - by posing and solving problems or asking questions about place, stories, and cultural practices
- **Explain and justify:**
 - using mathematical arguments to convince
 - includes anticipating consequences
- **decisions:**
 - Have students explore which of two scenarios they would choose and then defend their choice.
- **many ways:**
 - including oral, written, visual, use of technology
 - communicating effectively according to what is being communicated and to whom

Curricular Competencies – Elaborations

- **Represent:**
 - using models, tables, graphs, words, numbers, symbols
 - connecting meanings among various representations
- **discussions:**
 - partner talks, small-group discussions, teacher-student conferences
- **discourse:**
 - is valuable for deepening understanding of concepts
 - can help clarify students' thinking, even if they are not sure about an idea or have misconceptions
- **Reflect:**
 - share the mathematical thinking of self and others, including evaluating strategies and solutions, extending, posing new problems and questions
- **Connect mathematical concepts:**
 - to develop a sense of how mathematics helps us understand ourselves and the world around us (e.g., daily activities, local and traditional practices, popular media and news events, social justice, cross-curricular integration)
- **mistakes:**
 - range from calculation errors to misconceptions
- **opportunities to advance learning:**
 - by:
 - analyzing errors to discover misunderstandings
 - making adjustments in further attempts
 - identifying not only mistakes but also parts of a solution that are correct
- **Incorporate:**
 - by:
 - collaborating with Elders and knowledge keepers among local First Peoples
 - exploring the [First Peoples Principles of Learning](#) (e.g., Learning is holistic, reflexive, reflective, experiential, and relational [focused on connectedness, on reciprocal relationships, and a sense of place]; Learning involves patience and time)
 - making explicit connections with learning mathematics
 - exploring cultural practices and knowledge of local First Peoples and identifying mathematical connections
- **knowledge:**
 - local knowledge and cultural practices that are appropriate to share and that are non-appropriated
- **practices:**
 - [Bishop's cultural practices](#): counting, measuring, locating, designing, playing, explaining
 - [Aboriginal Education Resources](#)
 - [Teaching Mathematics in a First Nations Context](#), FNECS

Content – Elaborations

- **functions:**
 - parent functions from Pre-calculus 12
 - piecewise functions
 - inverse trigonometric functions
- **limits:**
 - from table of values, graphically, and algebraically
 - one-sided versus two-sided
 - end behaviour
 - intermediate value theorem
- **differentiation:**
 - history
 - definition of derivative
 - notation
- **rate of change:**
 - average versus instantaneous
 - slope of secant and tangent lines
- **differentiation rules:**
 - power, product; quotient and chain
 - transcendental functions: logarithmic, exponential, trigonometric
- **applications:**
 - relating graph of $f(x)$ to $f'(x)$ and $f''(x)$
 - increasing/decreasing, concavity
 - differentiability, mean value theorem
 - Newton's method
 - problems in contextual situations, including related rates and optimization problems
- **integration:**
 - definition of an integral
 - notation
 - definite and indefinite

Content – Elaborations

- **approximations:**
 - Riemann sum, rectangle approximation method, trapezoidal method
- **methods of integration:**
 - antiderivatives of functions
 - substitution
 - by parts
- **applications:**
 - area under a curve, volume of solids, average value of functions
 - differential equations
 - initial value problems
 - slope fields